Kinetic-molecular theory of gases

CH102 Spring 2013
Boston University

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Kinetic-molecular theory of gases

Goal: Relate *T* to speed of gas particles

Pathway: Get microscopic expression for *P V*

Key idea: Force is exchange of momentum *p* with wall per unit time.

Note: Here **upper-case *P*** is used for pressure and **lower-case *p*** is used for momentum.

Kinetic-molecular theory of gases

Force due to j^{th} particle of mass m and speed u_j

$$\Delta p = 2\ m\ u_j \text{ (elastic collision)}$$

$$\Delta t = 2\ L/u_j \text{ (travel to opposite wall and back)}$$

$$F = \Delta p/\Delta t = m\ u_j^2/L$$

Pressure due to j^{th} particle of mass m and speed u_j

$$P_j = F/\text{area} = F/L^2 = m\ u_j^2/L^3$$

That is

$$\boldsymbol{P_j = m\ u_j^2/V}$$

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Kinetic-molecular theory of gases

Pressure due to j^{th} particle of mass m and speed u_j

$$P_j = m\, u_j^2/V$$

Different particles have different speeds

In terms of the **average speed *u***, and adding up contributions of all of the particles in the gas, the **total pressure *P*** in terms of the number of moles ***n*** and the **molar mass *M*** is

$$P = n\, M\, u^2/(3\, V)$$

since the number of particles N times their mass m can be expressed as

$$N\, m = n\, N_o\, m = n\, M$$

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In terms of the **average molar kinetic energy, $E_{k,avg} = M u^2/2$** , the total pressure is

$$P = 2/3\ n\ E_{k,avg}/V$$

But from the ideal gas law

$$P = n\ R\ T/V$$

Combining these two expressions, we find that T is a measure of the average molar kinetic energy,

$$E_{k,avg} = (3/2)\ R\ T$$

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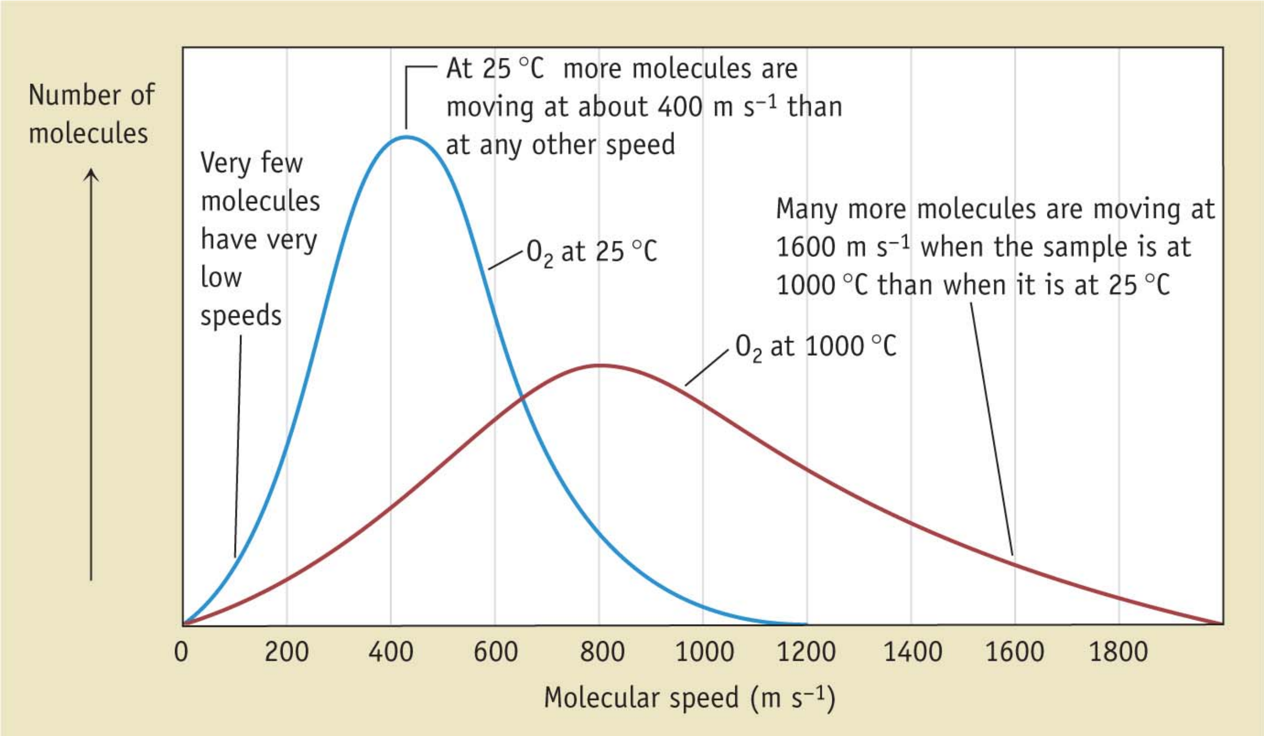
Kinetic-molecular theory of gases

T is a measure of the average molar kinetic energy,

$$E_{k,avg} = (3/2)\ R\ T$$

Since $M\ u^2/2$, the squared rms speed is …

$$u^2 = 3\ R\ T/M$$



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Calculate RMS speed of H_2 at 25 ˚C

$u^2 = 3\ R\ T/M$

$R = 8.314$ J/(K mol), J = kg m^2/s^2

Answer: **1920 m/s**

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Ratio of speed of H_2 to O_2 at 25 ˚C

Key relation …

$$u_1^2/u_2^2 = M_2/M_1$$

$$u_{H2}^2/u_{O2}^2 = M_{O2}/M_{H2} = 32/2 = 16$$

$$\boldsymbol{u_{O2} = u_{H2}/4 = (1920\ \text{m/s})/4 = \ldots}$$

480 m/s

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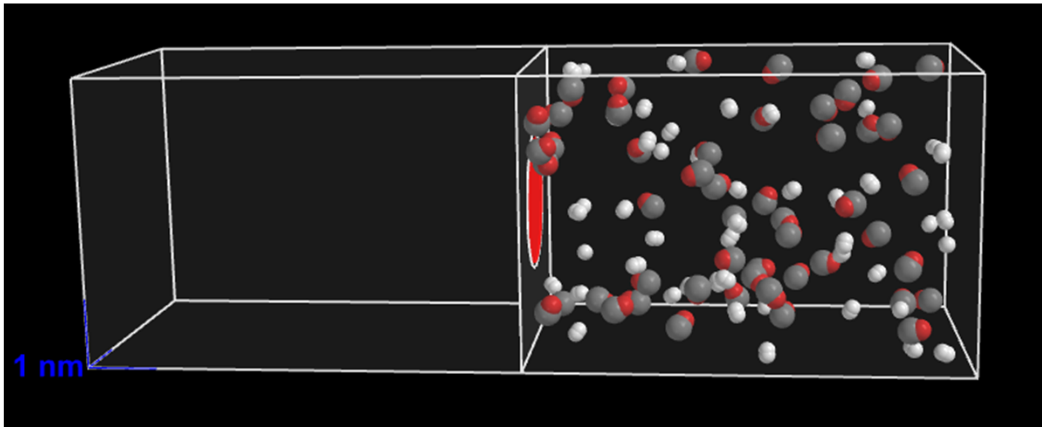
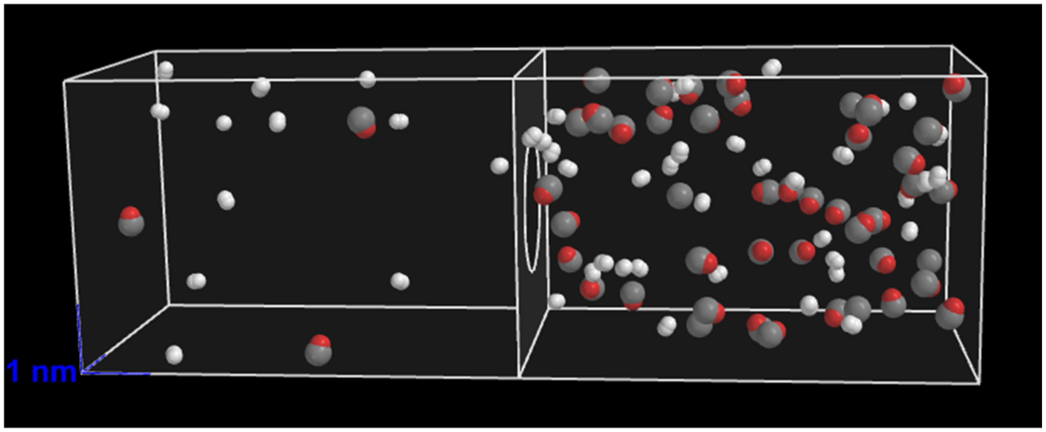
Effusion of gas mixtures

Odyssey Molecular Lab 34: CO and H_2





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Exercise 11.19: Graham's law of effusion

Key relation …

$$u_1^2/u_2^2 = M_2/M_1$$

If *n* moles of CH_4 effuses in 1.50 min and *n* moles of X effuses in 4.73 min, …

what is the expression for the molar mass of X?

Key idea …

time to effuse is proportional to 1/speed

$$t_2^2/t_1^2 = M_2/M_1$$

$$M_2 = M_1\, t_2^2/t_1^2$$

The molar mass of X is …

160. g/mol

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