

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

**[TP]** The enthalpy diagram shows changes associated with the reaction  
 $\text{Na}_2(g) + \text{Br}_2(g) \rightarrow 2 \text{NaBr}(g)$ .  
 The **uppermost horizontal line** corresponds to the species ...

0% 1.  $2 \text{Na}(s) + \text{Br}_2(l)$   
 0% 2.  $2 \text{Na}(g) + 2 \text{Br}(g)$   
 30% 3.  $\text{Na}_2(g) + \text{Br}_2(g)$   
 0% 4. Something else  
 70% 5. Not sure

**10 of 15**

1

Lecture 11 CH131 Summer 1 2021  
 Tuesday, June 15, 2021

- Complete: Enthalpy changes are additive: Hess's law
- Practice problem: Limiting reagent,  $\Delta n_{\text{gas}}$ ,  $w_{\text{PV}}$ ,  $q_{\text{P}}$ ,  $q_{\text{V}}$
- Standard enthalpy of formation,  $\Delta H^{\circ}_{\text{f}}$
- Using  $\Delta H^{\circ}_{\text{f}}$  to compute  $\Delta H^{\circ}_{\text{rxn}}$

Begin ch13: Spontaneous Processes

- The essence of change: Blind chance and dumb luck
- Arrangements (particles)  $\rightarrow$  entropy
- Heat (energy) flow  $\rightarrow$  entropy change
- Spontaneity of phase transitions

Next lecture: Continue chapter 13

2

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

$\Delta H^{\circ}_{\text{rxn}}$  via Hess's law

Consider

$$\begin{cases} \text{C}(s) + \text{O}_2(g) \rightarrow \text{CO}_2(g) & \Delta H_1 \\ \text{S}(s) + \text{O}_2(g) \rightarrow \text{SO}_2(g) & \Delta H_2 \\ \text{CS}_2(l) + 3 \text{O}_2(g) \rightarrow \text{CO}_2(g) + 2 \text{SO}_2(g) & \Delta H_3 \\ \text{C}(s) + 2 \text{S}(s) \rightarrow \text{CS}_2(l) & \Delta H_4 = ? \end{cases}$$

Since energy is conserved ...

$$\Delta H_4 = \Delta H_1 + 2 \Delta H_2 - \Delta H_3$$

11

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

$\Delta H^{\circ}_{\text{rxn}}$  via Hess's law

Consider

$$\begin{aligned} \text{C}(s) + \text{O}_2(g) &\rightarrow \text{CO}_2(g) & \Delta H_1 = -393.5 \text{ kJ} \\ \text{S}(s) + \text{O}_2(g) &\rightarrow \text{SO}_2(g) & \Delta H_2 = -296.8 \text{ kJ} \\ \text{CS}_2(l) + 3 \text{O}_2(g) &\rightarrow \text{CO}_2(g) + 2 \text{SO}_2(g) & \Delta H_3 = -1103.9 \text{ kJ} \\ \text{C}(s) + 2 \text{S}(s) &\rightarrow \text{CS}_2(l) & \Delta H_4 = ? \end{aligned}$$

Since energy is conserved ...

$$\begin{aligned} \Delta H_4 &= \Delta H_1 + 2 \Delta H_2 - \Delta H_3 = \dots \\ \Delta H_4 &= +116.8 \text{ kJ} \end{aligned}$$

12

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Recipe for $\Delta H^\circ_{\text{rxn}}$ via Hess's law

First, get expression for  $\Delta H^\circ_{\text{rxn}}$ .

Then, substitute in values in the expression for  $\Delta H^\circ_{\text{rxn}}$  to get its numerical value.

BOSTON UNIVERSITY

13

13

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Practice problem

$$2 \text{Al}(s) + 6 \text{HCl}(aq) \rightarrow 2 \text{AlCl}_3(aq) + 3 \text{H}_2(g) \quad \Delta H_1 = -1049 \frac{\text{kJ}}{\text{mol}} \checkmark$$

$$\text{HCl}(g) \rightarrow \text{HCl}(aq) \quad \Delta H_2 = -79 \frac{\text{kJ}}{\text{mol}} \checkmark$$

$$\text{H}_2(g) + \text{Cl}_2(g) \rightarrow 2 \text{HCl}(g) \quad \Delta H_3 = -185 \frac{\text{kJ}}{\text{mol}} \checkmark$$

$$\text{AlCl}_3(s) \rightarrow \text{AlCl}_3(aq) \quad \Delta H_4 = -323 \frac{\text{kJ}}{\text{mol}} \checkmark$$

Show that

$$2 \text{Al}(s) + 3 \text{Cl}_2(g) \rightarrow 2 \text{AlCl}_3(s) \quad \Delta H_{\text{rxn}} = -1432 \frac{\text{kJ}}{\text{mol}} \quad \textcircled{1}$$

BOSTON UNIVERSITY

14

14

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Practice problem

$$2 \text{Al}(s) + 6 \text{HCl}(aq) \rightarrow 2 \text{AlCl}_3(aq) + 3 \text{H}_2(g) \quad \Delta H_1 = -1049 \frac{\text{kJ}}{\text{mol}}$$

$$\text{HCl}(g) \rightarrow \text{HCl}(aq) \quad \Delta H_2 = -79 \frac{\text{kJ}}{\text{mol}} \quad \parallel$$

$$\text{H}_2(g) + \text{Cl}_2(g) \rightarrow 2 \text{HCl}(g) \quad \Delta H_3 = -185 \frac{\text{kJ}}{\text{mol}}$$

$$\text{AlCl}_3(s) \rightarrow \text{AlCl}_3(aq) \quad \Delta H_4 = -323 \frac{\text{kJ}}{\text{mol}}$$

$$2 \text{Al}(s) + 3 \text{Cl}_2(g) \rightarrow 2 \text{AlCl}_3(s) \quad \Delta H_{\text{rxn}} = \dots$$

Show that

$$2 \text{Al}(s) + 3 \text{Cl}_2(g) \rightarrow 2 \text{AlCl}_3(s) \quad \Delta H_{\text{rxn}} = -1432 \frac{\text{kJ}}{\text{mol}}$$

$$\Delta H_{\text{rxn}} = \Delta H_1 + 3\Delta H_3 - 2\Delta H_4 + 6\Delta H_2 = 1432 \frac{\text{kJ}}{\text{mol}} \checkmark$$

BOSTON UNIVERSITY

15

15

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Practice problem

$$2 \text{Al}(s) + 3 \text{Cl}_2(g) \rightarrow 2 \text{AlCl}_3(s) \quad \Delta H_{\text{rxn}} = -1432 \frac{\text{kJ}}{\text{mol}} \quad \textcircled{1}$$

Calculate  $q_p$  and  $q_v$  for the reaction of 1.350 g of Al(s) reacts with 0.7100 g of Cl<sub>2</sub>(g). Assume 100.0% yield. ✓

$q_p$  = heat at constant P  
for the actual amount consumed.

$q_v = q_p + w_{PV}$

$$\Delta H_{\text{rxn}} = 2 \Delta H_f^\circ(\text{AlCl}_3(s)) - 2 \Delta H_f^\circ(\text{Al}(s)) - 3 \Delta H_f^\circ(\text{Cl}_2(g))$$

BOSTON UNIVERSITY

16

16

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Practice problem

$$2 \text{Al}(s) + 3 \text{Cl}_2(g) \rightarrow 2 \text{AlCl}_3(s) \quad \Delta H_{\text{rxn}} = -1432 \frac{\text{kJ}}{\text{mol}}$$

Calculate  $q_P$  and  $q_V$  for the reaction of 1.350 g of Al(s) reacts with 0.7100 g of  $\text{Cl}_2(g)$ . Assume 100.0% yield.

Limiting reagent is ...

$$\Delta n_{\text{Cl}_2} = \dots \text{ mol}$$

$$w_{\text{PV}} = -\Delta n_{\text{Cl}_2} R T$$

$$q_P = \dots \text{ kJ}$$

$$q_V = q_P + w_{\text{PV}} = \dots \text{ kJ}$$

BOSTON UNIVERSITY

17

17

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Practice problem

$$2 \text{Al}(s) + 3 \text{Cl}_2(g) \rightarrow 2 \text{AlCl}_3(s) \quad \Delta H_{\text{rxn}} = -1432 \frac{\text{kJ}}{\text{mol}}$$

Calculate  $q_P$  and  $q_V$  for the reaction of 1.350 g of Al(s) reacts with 0.7100 g of  $\text{Cl}_2(g)$ . Assume 100.0% yield.

Limiting reagent is  $\text{Cl}_2$

$$\Delta n_{\text{Cl}_2} = -0.01001 \text{ mol}$$

$$w_{\text{PV}} = -\Delta n_{\text{Cl}_2} R T = -0.02482 \text{ kJ}$$

$$q_P = -4.780 \text{ kJ}$$

$$q_V = q_P + w_{\text{PV}} = -4.755 \text{ kJ}$$

BOSTON UNIVERSITY

18

18

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Standard enthalpy of formation, $\Delta H^\circ_f$

We have seen that using Hess's law, we can calculate the enthalpy change of a reaction in terms of enthalpy changes of other reactions.

$$\begin{array}{l} A \rightarrow B \quad \Delta H_1 = +85 \text{ kJ} \\ C \rightarrow B \quad \Delta H_2 = -52 \text{ kJ} \\ A \rightarrow C \quad \Delta H_3 = \Delta H_1 - \Delta H_2 = +85 \text{ kJ} - (-52 \text{ kJ}) = +137 \text{ kJ} \end{array}$$

To use this method systematically, a set of reference reactions is needed.

These reference reactions are called **formation reactions**.

BOSTON UNIVERSITY

19

19

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Standard enthalpy of formation, $\Delta H^\circ_f$

A **formation reaction** has just single product, one mole of substance X.

The **formation reaction** of substance X is defined as the **balanced chemical equation** having as the **single product** one mole of X, and as reactants the elements that X contains, each element in its **standard state**.

$$\text{H}_2(g) + \frac{1}{2} \text{O}_2(g) \rightarrow \text{H}_2\text{O}(l) \quad \Delta H_{\text{rxn}} \equiv \Delta H_f^\circ(\text{H}_2\text{O}, l)$$

→ X

BOSTON UNIVERSITY

20

20

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## Standard enthalpy of formation, $\Delta H^\circ_f$

The **standard state** of an element is its **most stable** form at SATP (1 bar and 25°C).

BOSTON UNIVERSITY

21

21

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## Standard enthalpy of formation, $\Delta H^\circ_f$

The **enthalpy change of reaction** of the **formation reaction** of substance X is defined as the **standard enthalpy of formation of substance X**,

$$\Delta H^\circ_f(X) \equiv \Delta H^\circ_{\text{rxn}}$$

for the formation of one mole of X from its elements

BOSTON UNIVERSITY

24

24

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## Standard enthalpy of formation, $\Delta H^\circ_f$

The **enthalpy change of reaction** of the **formation reaction** of substance X is defined as the **standard enthalpy of formation of substance X**,

$$\Delta H^\circ_f(X) \equiv \Delta H^\circ_{\text{rxn}}$$

for the formation of one mole of X from its elements

A consequence of this definition is that for each element in its standard state,

$$\Delta H^\circ_f(\text{element in its standard state}) = 0$$

BOSTON UNIVERSITY

25

25

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## What is the standard state at 25°C?

For example,

$$\text{O}_2(g) \rightarrow \text{O}_2(g), \Delta H^\circ_{\text{rxn}} = 0 = \Delta H^\circ_f(\text{O}_2, g)$$

This means we can use tabulations of standard enthalpies of formation to determine the standard state of an element.

BOSTON UNIVERSITY

26

26

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### What is the standard state at 25°C?

In tables of enthalpies of formation, such as <http://goo.gl/aljmi>, ...

Dinitrogen pentoxide	Gas	N <sub>2</sub> O <sub>5</sub>	11.3
<b>Oxygen</b>			
Monatomic oxygen	Gas	O	249
Oxygen	Gas	O <sub>2</sub>	0
Ozone	Gas	O <sub>3</sub>	143
<b>Phosphorus</b>			
White phosphorus	Solid	P <sub>4</sub>	0

the standard state will have value  $\Delta H_f^\circ = 0$ .

$\frac{1}{2} \text{O}_2(\text{g}) \rightarrow \text{O}(\text{g}) \quad \Delta H_{\text{rxn}} \equiv \Delta H_f^\circ(\text{O}, \text{g}) = +249 \text{ kJ/mol}$

$\Delta H_f^\circ(\text{O}_2, \text{g}) = 0$

BOSTON UNIVERSITY

27

27

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### $\Delta H_f^\circ$ of fructose, C<sub>6</sub>H<sub>12</sub>O<sub>6</sub>(s)

The **standard enthalpy of formation** of fructose is defined as ...  
the enthalpy change when **one mole of fructose** is formed ...  
from its elements, each in their **standard states**.

$$6 \text{C}(\text{s}) + 6 \text{H}_2(\text{g}) + 3 \text{O}_2(\text{g}) \rightarrow \text{C}_6\text{H}_{12}\text{O}_6(\text{s}) \quad \Delta H_{\text{rxn}} \equiv \Delta H_f^\circ(\text{C}_6\text{H}_{12}\text{O}_6, \text{s})$$

BOSTON UNIVERSITY

28

28

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### $\Delta H_f^\circ$ of fructose, C<sub>6</sub>H<sub>12</sub>O<sub>6</sub>(s)

Write down the balanced chemical equation whose **enthalpy change** is the **standard enthalpy of formation** of fructose, C<sub>6</sub>H<sub>12</sub>O<sub>6</sub>(s)

BOSTON UNIVERSITY

29

29

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### $\Delta H_f^\circ$ of fructose, C<sub>6</sub>H<sub>12</sub>O<sub>6</sub>(s)

Write down the balanced chemical equation whose **enthalpy change** is the **standard enthalpy of formation** of fructose, C<sub>6</sub>H<sub>12</sub>O<sub>6</sub>(s)

The enthalpy change,  $\Delta H_{\text{rxn}}$ , of the chemical reaction

$$6 \text{C}(\text{s}) + 6 \text{H}_2(\text{g}) + 3 \text{O}_2(\text{g}) \rightarrow \text{C}_6\text{H}_{12}\text{O}_6(\text{s})$$

is the standard enthalpy of formation of sugar,  $\Delta H_f^\circ[\text{C}_6\text{H}_{12}\text{O}_6, \text{s}]$ .

BOSTON UNIVERSITY

30

30



Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## Using $\Delta H^\circ_f$ to compute $\Delta H^\circ_{\text{rxn}}$

The **key feature** of standard **enthalpies of formation** is that, the enthalpy change of **any reaction**,  $\Delta H^\circ_{\text{rxn}}$ , can be computed using them, as ...

$$\Delta H^\circ_{\text{rxn}} = \text{Sum}[\Delta H^\circ_f(\text{products})] - \text{Sum}[\Delta H^\circ_f(\text{reactants})]$$


BOSTON UNIVERSITY 35

35

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## The essence of spontaneous change

Why does a drop of ink in water disperse?  
Why do salt water and fresh water mix?



BOSTON UNIVERSITY 65

65

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## The essence of spontaneous change

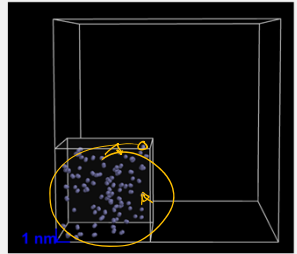
"things happen simply because they **can** happen  
and because they are statistically  
**most likely** to happen."  
Michael Munowitz, "Principles of Chemistry," W. W. Norton, 2000

BOSTON UNIVERSITY 66

66

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## A gas **fills** its container

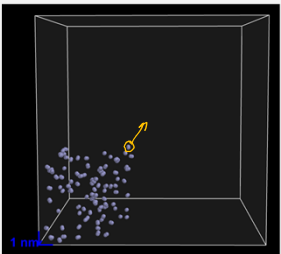


BOSTON UNIVERSITY 67

67

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### A gas fills its container



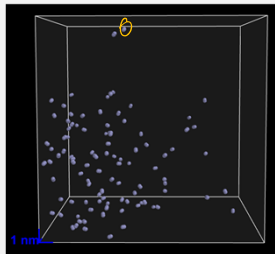
BOSTON UNIVERSITY

68

68

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### A gas fills its container



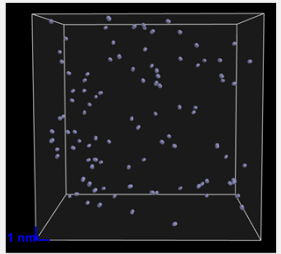
BOSTON UNIVERSITY

69

69

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### A gas fills its container




BOSTON UNIVERSITY

70

70

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### A gas fills its container



lattice gas

Gas all on left of container

BOSTON UNIVERSITY

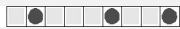
71

71



Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### A gas fills its container



Gas **evenly distributed** throughout container


BOSTON UNIVERSITY

72

72

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Pressure in a gas is uniform



moveable barrier

$$\frac{P_L}{RT} = \frac{M}{V} = \left(\frac{2}{8}\right) = \frac{1}{4}$$

$$\frac{P_R}{RT} = \frac{M}{V} = \left(\frac{6}{6}\right) = \frac{1}{1} = \frac{P_L}{RT}$$

$W_{iL} * W_{iR} = W_i$

$P$  proportional to  $n/V$  ("lattice gas")  
 $P$  higher on the right


BOSTON UNIVERSITY

73

73

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Pressure in a gas is uniform



moveable barrier

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{6}{12} = \frac{1}{2}$$

$W_{iL} * W_{iR} = W_i$

$P$  proportional to  $n/V$  ("lattice gas")  
 $P$  the same on the left and right

$W_i < W_f$


BOSTON UNIVERSITY

74

74

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Gases mix evenly



permeable barrier

One gas on left, another on right

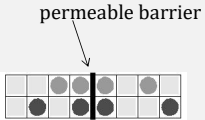
BOSTON UNIVERSITY

75

75

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## Gases **mix** evenly



permeable barrier

Equal amounts throughout

BOSTON UNIVERSITY

76

76

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## The essence of spontaneous change

The essential similarity in all of these behaviors...

- gas particles **spontaneously spread** uniformly throughout their container, so that pressure is everywhere the same;
- an ink drop **spontaneously disperses** throughout water;
- gases **spontaneously mix uniformly**, so that their partial pressures are everywhere the same;
- ...

BOSTON UNIVERSITY

77

77

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## The essence of spontaneous change

The essential similarity in all of these behaviors... is that **all spontaneous change** results in the **possible arrangements ( $W$ )** of particles that is **as large as it can be**:

$$W_i \rightarrow W_f = W_{\max}$$

BOSTON UNIVERSITY

78

78

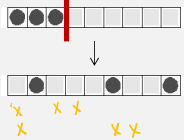
Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## The essence of spontaneous change

Spontaneous change **always results** in the **possible arrangements ( $W$ )** of particles that is **as large as it can be**.

For example,

gas particles **spontaneously spread** uniformly throughout their container, so that pressure is everywhere the same.



$$W_i = 1$$

$$W_f = \frac{(6+3)!}{6!3!} = 84$$

BOSTON UNIVERSITY

79

79

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## The essence of spontaneous change

“things happen simply because they **can** happen and because they are statistically **most likely** to happen.”

Michael Munowitz, "Principles of Chemistry," W. W. Norton, 2000

All **spontaneous change** results in the **possible arrangements ( $W$ )** of particles that is **as large as it can be**:

$$W_i \rightarrow W_f = W_{\max}$$


BOSTON UNIVERSITY 80

80

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## Arrangements $\rightarrow$ entropy

$$S = k_B \ln(W)$$

$$k_B = \frac{R}{N_A} = 1.4 \times 10^{-23} \text{ J/K}$$


BOSTON UNIVERSITY 81

81

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

$$S = k_B \ln(W)$$

Why natural log?

So that doubling the size of the system double the value of the spontaneity measure:

$$W \rightarrow W \times W = W^2$$

$$S \rightarrow k_B \ln(W^2) = 2k_B \ln(W) = 2S, \text{ so ...}$$

Boltzmann's definition in terms of natural log makes  $S$  **extensive** ...

$S$  scales with size of system

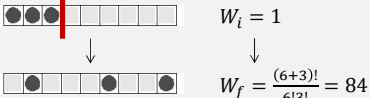
BOSTON UNIVERSITY 82

82

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

## The essence of spontaneous change

Gas particles **spontaneously spread** uniformly throughout their container, so that pressure is everywhere the same:



$$W_i = 1$$

$$W_f = \frac{(6+3)!}{6!3!} = 84$$

BOSTON UNIVERSITY 83

83

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

[TP] Using  $S = k_B \ln(W)$ , what is the value of  $\frac{\Delta S}{k_B}$  for  $W_i = 1 \rightarrow W_f = 84$ ?

0% 1. 1  
 100% 2. 4.4  
 0% 3. 83  
 0% 4. 84  
 0% 5. Something else  
 0% 6. Note sure

$\Delta S = S_f - S_i$   
 $= k_B \ln(W_f) - k_B \ln(W_i)$   
 $= k_B (\ln(W_f) - \ln(W_i))$   
 $= k_B \ln\left(\frac{W_f}{W_i}\right)$

If  $W_i < W_f$ ,  $\frac{W_f}{W_i} > 1$ ,  $\Delta S > 0$   
 If  $W_i > W_f$ ,  $\frac{W_f}{W_i} < 1$ ,  $\Delta S < 0$

$\frac{\Delta S}{k_B} = \ln\left(\frac{W_f}{W_i}\right) = \ln\left(\frac{84}{1}\right) = 391146$

13 of 15

84

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### The essence of spontaneous change

Gas particles spontaneously spread uniformly throughout their container, so that pressure is everywhere the same:

$W_i = 1$   
 $W_f = \frac{(6+3)!}{6!3!} = 84$

$\Delta S = S_f - S_i = k_B \ln\left(\frac{W_f}{W_i}\right) = k_B \ln\left(\frac{84}{1}\right) = k_B \times 4.4 > 0$

85

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Heat (energy) flow → entropy change

We can quantify the energy added to a system in terms of the number of small units (quanta, to analogous photons) of energy.

This means that adding energy increases the units of energy in the system.

And so, adding energy increases the ways,  $W_e$ , the energy can be stored in the system.

86

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Heat (energy) flow → entropy change

Just as spontaneous change corresponds to an increase in the number of arrangement of particles,  $W_p$ , spontaneous change also corresponds to an increase the ways energy is stored,  $W_e$ .

This means that added energy contributes to the entropy,  $S_e = k_B \ln(W_e)$ .

87

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Heat (energy) flow → entropy change

How does the entropy increase depend on how much energy is **already present**?

BOSTON UNIVERSITY 88

88

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

[TP] Temperature is a measure of how much (kinetic) energy is in the system.  
Which is larger?

$\Delta S_1$  for adding 10 J to 1 mol at 300 K *quiet library reading room*  
 $\Delta S_2$  for adding 10 J to 1 mol at 600 K? *noisy hockey game*

0% 1.  $\Delta S_1 < \Delta S_2$   
 0% 2.  $\Delta S_1 = \Delta S_2$   
 100% 3.  $\Delta S_1 > \Delta S_2$   
 0% 4. Not sure *two friends chatting.*

$2 \ln\left(\frac{W_{ef}}{W_{ei}}\right)_{\text{at } 300\text{K}} > \ln\left(\frac{W_{ef}}{W_{ei}}\right)_{\text{at } 600\text{K}}$   
*391146*

BOSTON UNIVERSITY 12 of 15 89

89

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Heat (energy) flow → entropy change

$\Delta S$  is larger **the less energy already present**

Temperature is a measure of how much (kinetic) energy is in the system.  
 This means that  $\Delta S$  is larger **the lower  $T$**  at which the energy is added and so that ...

$\Delta S \sim \frac{1}{T}$

BOSTON UNIVERSITY 90

90

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Heat (energy) flow → entropy change

How does the entropy increase depend on how much energy is **added**?

BOSTON UNIVERSITY 91

91

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

[TP] The more energy added, the more quanta of energy there are in the system. Which is larger?

$\Delta S_1$  for adding 10 J to 1 mol at 400 K ||

$\Delta S_2$  for adding 20 J to 1 mol at 400 K ||

100% 1.  $\Delta S_1 < \Delta S_2$

0% 2.  $\Delta S_1 = \Delta S_2$

0% 3.  $\Delta S_1 > \Delta S_2$

0% 4. Not sure

39/146

11 of 15

BOSTON UNIVERSITY

92

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Heat (energy) flow $\rightarrow$ entropy change

The more energy added, the more quanta of energy there are in the system and so the larger the number arrangements,  $W_e$ .

This means  $\Delta S$  is larger the more energy added and so that ...

$$\Delta S \sim \Delta H$$

BOSTON UNIVERSITY

93

93

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Heat (energy) flow $\rightarrow$ entropy change

$\Delta S$  is larger the more energy added

$$\Delta S \sim \Delta H$$

$\Delta S$  is larger the lower  $T$  at which the energy is added.

$$\Delta S \sim \frac{1}{T}$$

Amazingly, these results combine into the fundamental relation for entropy change due to energy flow,

$$\Delta S = \frac{\Delta H}{T}$$

$W_P$   
 $W_e$

BOSTON UNIVERSITY

94

94

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Spontaneity = increase in total entropy

Spontaneity means that what happens is always associated with an increase in entropy,  $\Delta S > 0$ .

We will see that it is useful to partition entropy change as that due to the chemical or physical system (what we are interested in) and the surroundings (everything else).

BOSTON UNIVERSITY

95

95

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Spontaneity = increase in total entropy

Therefore, the spontaneity condition can be written as

$$\Delta S_{tot} = \Delta S_{sys} + \Delta S_{sur} > 0$$

Note that this requirement **does not** require separately that ...

$$\Delta S_{sys} > 0 \text{ or } \Delta S_{sur} > 0$$

BOSTON UNIVERSITY 96

96

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

### Spontaneity = increase in total entropy

A neat illustration of the **roles** of  $\Delta S_{sys}$  and  $\Delta S_{sur}$  is understanding why **steam condenses** and **water boils**.

Water **spontaneously** ( $\Delta S_{tot} > 0$ ) vaporizes when  $T > 100^\circ\text{C}$

Water and steam are in **equilibrium** ( $\Delta S_{tot} = 0$ ) when  $T = 100^\circ\text{C}$

Steam **spontaneously** ( $\Delta S_{tot} > 0$ ) condenses when  $T < 100^\circ\text{C}$

BOSTON UNIVERSITY 97

97

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

[TP] For **steam**  $\rightarrow$  **liquid**, the system is the water (**liquid and steam**), and the surroundings is everything else. For **steam**, which of the following **must** always be true, **independently of temperature**?

- 0% 1.  $\Delta S_{sys} < 0$  and  $\Delta S_{sur} < 0$
- 0% 2.  $\Delta S_{sys} < 0$  and  $\Delta S_{sur} = 0$
- 100% 3.  $\Delta S_{sys} < 0$  and  $\Delta S_{sur} > 0$
- 0% 4.  $\Delta S_{tot} < 0$
- 0% 5.  $\Delta S_{tot} = 0$
- 0% 6.  $\Delta S_{tot} > 0$
- 0% 7. None of the above
- 0% 8. Not sure

$\text{H}_2\text{O}(g) \rightarrow \text{H}_2\text{O}(l)$

$\Delta S_{sys} < 0$

$\Delta S_{sur} = \frac{\Delta H_{sur}}{T}$ ,  $\Delta H_{sur} > 0$

$\Delta H_{sur} > 0$

391146

BOSTON UNIVERSITY 13 of 15

98

Lecture 11 CH131 Summer 1 2021 Copyright © 2021 Dan Dill dan@bu.edu

**steam**  $\rightarrow$  **liquid water at 94°C**

Since  $\Delta S_{sys} < 0$  and  $\Delta S_{sur} > 0$ , their contributions to

$$\Delta S_{tot} = \Delta S_{sys} + \Delta S_{sur}$$

oppose one another.

$\text{H}_2\text{O}(g) \rightarrow \text{H}_2\text{O}(l)$

$\Delta S_{sys} < 0$

$\Delta H_{sur} > 0, \Delta S_{sur} = \frac{\Delta H_{sur}}{T} > 0$

BOSTON UNIVERSITY 99

99