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## Lecture 6 CH131 Summer 12021 <br> Thursday, June 3, 2021

- Kinetic molecular theory
- Why the gas law does not depend on mass
- Distribution of molecular speeds
- Real gases
- Gas law for real gases: van der Waals equation

Next lecture: Ch10: Solids, liquids and phase transitions

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## Kinetic molecular theory

If we assume particles of a gas collide with one another and with the walls of the container elastically, then we can show that the contribution of a single


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## Kinetic molecular theory

If we assume particles of a gas collide with one another and with the walls of the container elastically, then we can show that the contribution of a single particle $i$ to the pressure of a gas is

$$
P_{i}=\frac{1}{3} \frac{m v_{i}^{2}}{V}
$$

Note that the units of this expression, $\frac{\text { energy }}{\text { volume }}$, are correct for pressure.

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## Kinetic molecular theory

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Note that the units of this expression, $\frac{\text { energy }}{\text { volume }}$, are correct for pressure.
The factor $\frac{1}{3}$ takes into account that motion takes place in three dimensions.

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## Kinetic molecular theory

The total pressure is the sum of the contributions of the $N=n N_{\mathrm{A}}$ particles of the gas,

$$
P=P_{1}+P_{2}+\cdots+P_{N}=\frac{1}{3} \frac{m}{V}\left(v_{1}^{2}+v_{2}^{2}+\cdots+v_{N}^{2}\right)
$$

We can simplify this expression by dividing the sum of squared speeds by the number of particles ( $N$ ),

$$
P=\frac{1}{3} \frac{m N}{V}\left(\frac{v_{1}^{2}+v_{2}^{2}+\cdots+v_{N}^{2}}{N}\right)=\frac{1}{3} \frac{m N}{V} v_{r m s}^{2}
$$

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## Kinetic molecular theory

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$$

The quantity $v_{\text {rims }}^{2}=\frac{v_{1}^{2}+v_{2}^{2}+\cdots+v_{N}^{2}}{N}$ is the mean (average) of the squared speeds of the individual particles.
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## Kinetic molecular theory

The square root of the mean of the squared speeds is known the root-meansquared speed,

$$
v_{r m s}=\sqrt{\frac{3 R T}{M}}
$$

$\qquad$

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## Kinetic molecular theory

Comparing the expression for pressure from the kinetic molecular theory

$$
P=\frac{1}{3} \frac{m N}{V} v_{r m s}^{2}=\frac{1}{3} \frac{m N_{\mathrm{A}} n}{V} v_{r m s}^{2}=\frac{1}{3} \frac{M n}{V} v_{r m s}^{2}
$$

to that from the ideal gas law,

$$
P=\frac{n R T}{V}
$$

we get the fundamental connection between temperature and the mean of the // squared speeds,

$$
v_{r m s}^{2}=\frac{3 R T}{M} / \int
$$

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[TP] The rms speed of $\mathrm{O}_{2}$ at $25^{\circ} \mathrm{C}$ is about $500 \mathrm{~m} / \mathrm{s}$. Without using a calculator,
use the rms speed of $\mathrm{O}_{2}$ to estimate that the rms speed of $\mathrm{H}_{2}$ at the same
temperature is about

$$
N_{\mathrm{H}_{2}}=4 N_{\mathrm{O}_{\mathrm{z}}}
$$



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## Why the gas law does not depend on mass

We have seen that the microscopic model of gas pressure is that
/ $P=\frac{1}{3} \frac{M n}{V} v_{r m s}^{2}$ where $v_{r m s}=\sqrt{\frac{3 R T}{M}}$,
Why doesn't $P$ depend on molar mass $(M)$ ?
The reason is that root-mean-square speed offsets the apparent mass dependence,

$$
P=\frac{1}{3} \frac{M n}{V} v_{r m s}^{2}=\frac{1}{3} \frac{M n}{V} \times \frac{3 R T}{M}=\frac{n R T}{V}
$$



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## Distribution of molecular speeds Maxuel(-Boltzмама

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Here is the distribution of speeds of $\mathrm{O}_{2}$ at 300 K , from $0 \mathrm{~m} / \mathrm{s}$ to $4000 \mathrm{~m} / \mathrm{s}$, marking the position of $v_{r m s}$ on the $x$ axis.
The lowest rms speed is 0 and speeds extend exponentially to high values.


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## $v_{\text {rms }}$ versus temperature

The rms speed of $\mathrm{O}_{2}$ at 300 K is $500 \mathrm{~m} / \mathrm{s}$ and the rms speed of $\mathrm{O}_{2}$ at 1200 K is $1000 \mathrm{~m} / \mathrm{s}$. Sketch the distribution of speeds for $\mathrm{O}_{2}$ at these two temperature on the same axes, marking the position of the two rms speeds on the $x$ axis.

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## $v_{\text {rms }}$ versus temperature

The rms speed of $\mathrm{O}_{2}$ at 300 K is $500 \mathrm{~m} / \mathrm{s}$ and the rms speed of $\mathrm{O}_{2}$ at 1200 K is $1000 \mathrm{~m} / \mathrm{s}$. Sketch the distribution of speeds for $\mathrm{O}_{2}$ at these two temperatures on the same axes, marking the position of the two rms speeds on the $x$ axis.


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[TP] Use the rms speed of $\mathrm{O}_{2}$ at $300 \mathrm{~K}, v_{r m s}=500 \mathrm{~m} / \mathrm{s}$. Without a calculator, use this value to find the rms speed of $\mathrm{H}_{2}$ at 300 K .

## $v_{\text {rms }}$ versus temperature

Add to your sketch the distribution of speeds of $\mathrm{H}_{2}$ at 300 K .

| $0 \%$ | 1. | $4000 \mathrm{~m} / \mathrm{s}$ |
| ---: | :--- | :--- |
| $100 \%$ | 2. | $2000 \mathrm{~m} / \mathrm{s}$ |
| $0 \%$ | 3. | $1000 \mathrm{~m} / \mathrm{s}$ |
| $0 \%$ | 4. | $500 \mathrm{~m} / \mathrm{s}$ |
| $0 \%$ | 5. | $250 \mathrm{~m} / \mathrm{s}$ |
| $0 \%$ | 6. | Not sure |



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## Real gases ||

While the ideal gas law is the same for every gas, there are two effects in all real gases that are important at high gas densities (due to temperatures close to condensation or very high pressures)

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$$
\frac{0.000614 \mathrm{~L}}{1 \mathrm{~L}} \times 100 \%=0.06 \% \text { Mot empty space }
$$

of the 1-L volume is taken up by the water molecules,


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## Real gases

More generally, the volume of the container is expressed as empty space
( $V_{\text {ideal }}$ ) plus that taken up by the molecules themselves,
$V=V_{\text {ideal }}$
0.0006 L

Here the parameter van der Waals $b$ is the volume taken up by one mole of the molecules of the substance.

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## Real gases

The second effect is due to attraction of the gas particles for one another.
This attraction lessens the impact of each particle with the container walls and so reduces the pressure.


## Real gases

The reduction in pressure is proportional to the number of particles hitting the wall and the number of neighboring particles lessening each particle's impact with the wall,

$$
P=P_{\text {ideal }}-a\left(\frac{n}{V}\right)^{2}
$$

The parameter van der Waals $a$ is a measure of the intermolecular attraction exerted between particles of the gas.

## Real gases

Computations with the van der Waals equation are usually done by rearranging it for the observed pressure,
$P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2}$
and using tabulated values of van der Waals $a$ and $b$.

| Gas | Formula | $a / L^{2} \cdot$ bar ${ }^{\text {mol }}{ }^{2}$ | b/L-mol ${ }^{-1}$ |
| :---: | :---: | :---: | :---: |
| ammonia | $\mathrm{NH}_{3}$ | 4.3044 | 0.037847 |
| carbon dioxide | $\mathrm{CO}_{2}$ | 3.6551 | 0.042816 |
| methane | $\mathrm{CH}_{4}$ | 2.3026 | 0.043067 |
| neon | Ne | 0.2167 | 0.017383 |
| nitrogen | $\mathrm{N}_{2}$ | 1.3661 | 0.038577 |
| oxygen | $\mathrm{O}_{2}$ | 1.3820 | 0.031860 |
| propane | $\mathrm{C}_{3} \mathrm{H}_{8}$ | 9.3919 | 0.090494 |

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## Real gases

The ideal gas law relates $P_{\text {ideal }}$ and $V_{\text {ideal }}$,

$$
P_{\text {ideal }} V_{\text {ideal }}=n R T .
$$

Using the expressions $V=V_{\text {ideal }}+b n$ and $P=P_{\text {ideal }}-a\left(\frac{n}{V}\right)^{2}$, we can rewrite the ideal gas law in terms of actual pressure and the actual container volume,

$$
\left(P+a\left(\frac{n}{V}\right)^{2}\right)(V-b n)=n R T>P P+a\left(\frac{x}{V}\right)^{2}=\frac{M R T}{V-b x}
$$

This is known as the van der Waal equation.

$$
\begin{aligned}
& P_{\text {ideal }}=P+a\left(\frac{M}{V}\right)^{2} \\
& V_{\text {ideal }}=V-b M n \\
& P_{\text {ideal }}-a\left(\frac{n}{V}\right)^{2}, \text { we can rewrite }
\end{aligned}
$$

## Real gases

Computations with the van der Waals equation are usually done by rearranging it for the observed pressure,

$$
P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2}
$$

This expression makes it easier to see that $a$ reduces pressure and $b$ increases pressure, relative to ideal values.

## Working with the van der Waals equation

$$
P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2} \text { and } R=0.08314 \mathrm{~L} \text { bar } /(\mathrm{K} \mathrm{~mol})
$$

For $\mathrm{Cl}_{2}, a=6.58$ bar $\mathrm{L}^{2} / \mathrm{mol}^{2}$ and $b=0.0562 \mathrm{~L} / \mathrm{mol}$. What are the ideal and observed pressures of 3.00 mol of $\mathrm{Cl}_{2}$ confined in 4.00 L at 500 K ?

## Working with the van der Waals equation

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P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2} \text { and } R=0.08314 \mathrm{~L} \text { bar } /(\mathrm{K} \mathrm{~mol})
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For $\mathrm{Cl}_{2}, a=6.58 \mathrm{bar} \mathrm{L}^{2} / \mathrm{mol}^{2}$ and $b=0.0562 \mathrm{~L} / \mathrm{mol}$. What are the ideal and observed pressures of 3.00 mol of $\mathrm{Cl}_{2}$ confined in 4.00 L at 500 . K ?

$$
P_{\text {ideal }}=\frac{n R T}{V}=31.3 \operatorname{bar}(\text { confirm yourself })
$$

## Working with the van der Waals equation

$$
P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2} \text { and } R=0.08314 \mathrm{~L} \text { bar } /(\mathrm{K} \mathrm{~mol})
$$

For $\mathrm{Cl}_{2}, a=6.58$ bar $\mathrm{L}^{2} / \mathrm{mol}^{2}$ and $b=0.0562 \mathrm{~L} / \mathrm{mol}$. What are the ideal and observed pressures of 3.00 mol of $\mathrm{Cl}_{2}$ confined in 4.00 L at $500 . \mathrm{K}$ ?
$P_{\text {ideal }}=\frac{n R T}{V}=31.3 \mathrm{bar}$ (confirm yourself)
$P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2}=32.5 \mathrm{bar}-3.7 \mathrm{bar}=28.8 \mathrm{bar}$ (confirm yourself)
Since the observed pressure is smaller than the ideal pressure, the effect of $a$ is more important than $b$ at 500. K.

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## Working with the van der Waals equation

$$
P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2} \text { and } R=0.08314 \mathrm{~L} \mathrm{bar} /(\mathrm{K} \mathrm{~mol})
$$

For $\mathrm{Cl}_{2}, a=6.58$ bar $\mathrm{L}^{2} / \mathrm{mol}^{2}$ and $b=0.0562 \mathrm{~L} / \mathrm{mol}$. What are the ideal and observed pressures of 3.00 mol of $\mathrm{Cl}_{2}$ confined in 4.00 L at 3000 . K ?

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## Working with the van der Waals equation

$$
p=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2} \text { and } R=0.08314 \mathrm{~L} \text { bar } /(\mathrm{K} \mathrm{~mol})
$$

For $\mathrm{Cl}_{2}, a=6.58$ bar $\mathrm{L}^{2} / \mathrm{mol}^{2}$ and $b=0.0562 \mathrm{~L} / \mathrm{mol}$. What are the ideal and observed pressures of 3.00 mol of $\mathrm{Cl}_{2}$ confined in 4.00 L at 3000 . K ?

$$
\begin{aligned}
& P_{\text {ideal }}=\frac{n R T}{V}=187.1 \text { bar } \text { (confirm yourself) } \\
& P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2}=195.3 \mathrm{bar}-3.7 \mathrm{bar}=191.6 \text { bar (confirm yourself) }
\end{aligned}
$$

Since the observed pressure is larger than the ideal pressure, the effect of $b$ is more important than $a$ at 3000 . K.

## Working with the van der Waals equation

$$
P=\frac{n R T}{V-b n}-a\left(\frac{n}{V}\right)^{2} \text { and } R=0.08314 \mathrm{~L} \text { bar } /(\mathrm{K} \mathrm{~mol})
$$

In general, the effect of $b$ is muted at low $T$ and so the effect of $a$ dominates there, while for high $T$ the effect of $b$ dominates.

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