[TP] Consider 1 mol each of two different gases, A and B, each with the same volume ( 1 L ) and temperature $\left(25^{\circ} \mathrm{C}\right)$. The molar mass of A is twice the molecular mass of $B$. What is the relationship between the pressures of $A$ and B?

$$
\begin{array}{rll}
89 \% & 1 . & P_{A}=2 P_{B} \\
0 \% & 2 . & P_{A}=\frac{P_{B}}{2} \\
0 \% & 3 . & P_{A}=4 P_{B} \\
0 \% & 4 . & P_{A}=\frac{P_{B}}{4} \\
0 \% & 5 . & P_{A}=P_{B} \\
11 \% & 6 . & \text { Not sure }
\end{array}
$$

## Ideal gas law

In the expression $P=c^{\prime \prime} n T / V$, the remaining constant of proportionality, $\left(c^{\prime \prime}\right)$, is known as the gas constant $R, P=R n T / V$.

This expression is written as

$$
P V=n R T
$$

and it is known as the ideal gas law.
The adjective "ideal" signifies that this relation does not depend of what the gas particles are.

## oftrsin

## Lecture 5 CH131 Summer 12021

Wednesday, June 2, 2021

- Ideal gas law
- Gas constant $R$, STP, and SATP
- Gas law calculational recipe
- Gas density

Dalton's law of partial pressures

- Gas law calculations in chemical reactions

Next lecture: Kinetic molecular theory; Why the gas law does not depend on mass; Distribution of molecular speeds; Real gases; Gas law for real gases: van der Waals equation; Ch10: Solids, liquids and phase transitions

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[TP] Consider 1 mol each of two different gases, $A$ and $B$, each with the same volume ( 1 L ) and temperature $\left(25^{\circ} \mathrm{C}\right)$. The molar mass of A is twice the molecular mass of $B$. What is the relationship between the pressures of $A$ and B?

$$
\begin{array}{lll}
8 \% & 1 . & P_{A}=2 P_{B} \\
8 \% & 2 . & P_{A}=\frac{P_{B}}{2}
\end{array}
$$

$$
0 \% \text { 3. } \quad P_{A}=4 P_{B}
$$

$$
0 \% \text { 4. } \quad P_{A}=\frac{P_{B}}{4}
$$

$$
85 \% 5 . \quad P_{A}=P_{B}
$$

$0 \% 6$. Not sure
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## Ideal gas law

The ideal gas law

$$
P V=n R T
$$

is as true for a gas of $\mathrm{O}_{2}$ molecules as for a gas of Xe atoms.

## Bostrin

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## Gas constant $R$

$$
\begin{aligned}
& \text { Often, pressures are measured in atm (1.01325 bar) and volumes are } \\
& \text { measured in } \mathrm{L}\left(1 \mathrm{~L}=10^{-3} \mathrm{~m}^{3}\right) \text {. } \\
& \text { Since } 1 \mathrm{~m}^{3}=1000 \mathrm{~L} \text { and } 1 \mathrm{~Pa}=1 \mathrm{~atm} / 101325, / / \\
& 1 \mathrm{~L}=\mathrm{m}^{3} \mathrm{~Pa}=1000 \mathrm{~L} \times \frac{\mathrm{atm}}{\underline{101325}}={ }^{0.00986923} \mathrm{~L} \text { atm } \\
& P=\frac{\text { enenge }}{\text { coleme }} \\
& \text { enerys }=P \text { noluence } \\
& \text { From thiswe can evaluate that } \\
& \left.\stackrel{R}{=}=\underline{8.314472} \frac{\mathrm{~J}}{\mathrm{~K} \mathrm{~mol}} \times \frac{0.00986923 \mathrm{Latm}}{\mathrm{~J}}=\underline{0.0820575} \frac{\mathrm{Latm}}{\mathrm{~K} \mathrm{~mol}}\right)
\end{aligned}
$$

Gas constant $R \quad P V=M R T \Rightarrow R=\frac{P_{1} V_{1}}{M} \quad \begin{aligned} & T \\ & \text { The value of the gas constant has been determined by measuring } V \text { for a given }\end{aligned}$ $P, n$, and $T$.
For example, the volume of 1 mol of gas at $20^{\circ} \mathrm{C}$ and $1 \mathrm{bar}=100000 \mathrm{~Pa}=$ $100000 \mathrm{~J} / \mathrm{m}^{3}$ is
$V=0.02437387 \mathrm{~m}^{3}$ II
From this we can evaluate that


10

## Gas constant $R$

$$
1 \mathrm{~J}=1 \mathrm{~Pa} \mathrm{~m}^{3}=10^{-5} \mathrm{bar} \times 10^{3} \mathrm{~L}=10^{-2} \mathrm{~L} \text { bar }
$$

an alternative expression for the value of the gas constant

$$
R=8.314472 \frac{\mathrm{~J}}{\mathrm{~K} \mathrm{~mol}}
$$

is

$$
R=\underline{0.08314472 \frac{\mathrm{Lbar}}{\mathrm{~K} \mathrm{~mol}}}
$$

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## Gas constant $R$

$$
\left.\begin{array}{l}
R=8.314472 \frac{\mathrm{~J}}{\mathrm{~K} \mathrm{~mol}} \\
R=0.08314472 \frac{\mathrm{~L} \mathrm{bar}}{\mathrm{~K} \mathrm{~mol}}
\end{array}\right)
$$

Finally, pressures are sometimes measured in atm (1.01325 bar) and so a third expression for the value of the gas constant is

$$
\begin{aligned}
& \text { pression for the value of the gas constant is } \\
& R=0.08314472 \frac{\mathrm{~L} \mathrm{bar}}{\mathrm{~K} \mathrm{~mol}} \times \frac{\mathrm{atm}}{1.01325 \mathrm{bar}}=0.0820575 \frac{\mathrm{~L} \mathrm{~atm}}{\mathrm{~K} \mathrm{~mol}}
\end{aligned}
$$

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## Gas constant $R$

$$
\begin{aligned}
& R=8.314472 \frac{\mathrm{~J}}{\mathrm{~K} \mathrm{~mol}} \| \\
& R=0.08314472 \frac{\mathrm{Lbar}}{\mathrm{~K} \mathrm{~mol}} \| \\
& R=0.0820575 \frac{\mathrm{Latm}}{\mathrm{~K} \mathrm{~mol}} \|
\end{aligned}
$$

In calculations, use the value of $R$ with the most convenient units.

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## STP and SATP

Two different "standard" values of temperature and pressure are defined.
STP is Standard Temperature and Pressure,

$$
T=273.15 \mathrm{~K}=0^{\circ} \mathrm{C} \text { and } P=1 \mathrm{bar}=10^{5} \mathrm{~Pa}
$$

$\mid$ SATP is Standard $\overline{\text { Ambient Temperature and Pressure }}$
$T=298.15 \mathrm{~K}=25^{\circ} \mathrm{C}$ and $P=1 \mathrm{bar}=10^{5} \mathrm{~Pa}$

## STP and SATP

What is the volume (in L ) of 1.00 mol of an ideal gas at standard ambient temperature and pressure, SATP?

$$
V=\frac{n R T}{P}=\frac{1 \mathrm{~mol} 0.08314 \mathrm{~L} \mathrm{bar} /(\mathrm{K} \mathrm{~mol}) 298.15 \mathrm{~K}}{1.00 \mathrm{bar}}=24.5 \mathrm{~L} \quad \text { STP } 22 \mathrm{~L} \quad \text { SATP } 24.5 \mathrm{~L}
$$

## Gas law calculational recipe

The recipe is to rearrange $P V=n R T$ so that what does not change is grouped on one side of the equality and does change is grouped on the other.

## Gas law calculational recipe

A 2-liter cylinder contains a gas ot 250 . lPa at $18^{\circ} \mathrm{C}$. What will be the pressure at $125^{\circ} \mathrm{C}$ ?

The volume $(V)$ and amount $(n)$ of gas are constant, so rearrangement gives
|| $\frac{n R}{V}=\frac{P_{1}}{T_{1}}=\frac{P_{2}}{T_{2}} \rightarrow P_{2}=P_{1} \frac{T_{2}}{T_{1}}$
$P_{2}=250 . \mathrm{kPa} \times \frac{(273.15+125) \mathrm{K}}{(273.15+18) \mathrm{K}}=342 \mathrm{kPa} \|$

## Gas law calculational recipe

A- 2 liter cylinder contains a gas at 250 kPa at $18^{\circ} \mathrm{C}$. What will be the pressure at $125^{\circ} \mathrm{C}$ ?
Heating the gas makes the particles move faster, there will be more collisions with the container walls in a given time, and so the pressure will increase.

## Gas law calculational recipe

A piston containing $25.0 \mathrm{dm}^{3}$ of gas at $45^{\circ} \mathrm{C}$ is expanded to $55.0 \mathrm{dm}^{3}$.
The increase in volume means the gas particles must travel farther between collisions with the container walls, there will be fewer collisions with the walls in a given time, and so the pressure will decrease.
The original pressure can be restored by heating the gas so that the noreased $V_{2}=55.0^{2}$ speed offsets the greater distance.
To what temperature must the gas be heated to restore the pressur to before the expansion?

Gas law calculational recipe
A piston containing $25.0 \mathrm{dm}^{3}$ of gas at $45^{\circ} \mathrm{C}$ is expanded to $55.0 \mathrm{dm}^{3}$. To what
2 temperature must the gas be heated to maintain the same pressure as before the expansion?
corstant
The pressure $(P)$ and amount $(n)$ of gas are constant, so rearrangeinent gives
$\frac{n R}{P}=\frac{V_{1}}{T_{1}}=\frac{V_{2}}{T_{2}} \rightarrow T_{2}=T_{1} \frac{V_{2}}{V_{1}}$
$T_{2}=(273.15+45) \mathrm{K} \times \frac{55.0 \mathrm{dm}^{3}}{25.0 \mathrm{dm}^{3}}=700 . \mathrm{K}=427^{\circ} \mathrm{C}$


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## Gas law calculations

Calculate the density of water in the gas phase at $100.0^{\circ} \mathrm{C}$ and 1.00 atm .
Rearrangement of the ideal gas law gives


## Gas law calculations

Calculate the density of water in the gas phase at $100.0^{\circ} \mathrm{C}$ and 1.00 atm .
Rearrangement of the ideal gas law gives

$$
\begin{aligned}
& \frac{n}{V}=\frac{P}{R T}=\frac{1.00 \mathrm{~atm}}{0.0820575 \frac{\mathrm{Latm}}{\mathrm{Kamol}} \times 373.25 \mathrm{~K}}=0.0327 \frac{\mathrm{~mol}}{\mathrm{~L}} \\
& \frac{n}{V}=0.0327 \frac{\mathrm{~mol}}{\mathrm{~L}} \times \frac{(2 \times 1.008+15.999) \mathrm{g}}{\mathrm{~mol}} \times \frac{\mathrm{L}}{1000 \mathrm{~mL}}=0.000588 \frac{\mathrm{~g}}{\mathrm{~mL}}
\end{aligned}
$$

## Gas law calculations

At $100.0^{\circ} \mathrm{C}$, the density of water vapor is $0.000588 \mathrm{~g} / \mathrm{mL}$ and that of liquid water is $0.958 \mathrm{~g} / \mathrm{mL}$. How much farther apart of water molecules in the gas phase than those in the liquid phase?

The ratio of the densities,
 0.758 g
how many more liquid water molecules there are in a given volume than there are gaseous water molecules in,the same volume.

> er molecules in the e same volume. a lot mave $\frac{H_{2} O l e s}{1630 H_{2}(l e)} / 1 H_{2} O(g)$

## Gas law calculations

This means the same volume can hold 1630 times as many liquid water molecules as gaseous water molecules.
Since volume is the cube of distance, the cube root of the ratio,

means the gas particles are about 11.8 times farther apart than liquid particles
So, about a factor of ten farther apart than in liquids or solids.

## Partial pressures

A convenient way to calculate partial pressures is to use mole fractions, $x_{a}=\frac{n_{a}}{n}, x_{b}=\frac{n_{b}}{n}, \ldots$
This means each kind of gas contributes to the total pressure only in proportion to its relative number of moles

$$
P=\frac{n_{b} R T}{V}+\frac{n_{b} R T}{V}+\cdots=P_{a}+P_{b}+\cdots
$$

where $P_{a}, P_{b}, \ldots$, are the partial pressures of gas $a$, gas $b$, etc., $P_{a}=\frac{n_{a} R T}{V}$, etc.

## Partial pressures

We have seen that the ideal gas law is the same for all gases, since it depends only on the number (moles) of particles but not what kind of particles they are

$$
x_{a}=\frac{n_{a}}{n}, x_{b}=\frac{n_{b}}{n}, \ldots
$$



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[TP] When 1 mol of $\mathrm{H}_{2}(g)$ is added to a cylinder of volume $V$ and at temperature $T$ the pressure is 1 bar . Then an additional 0.5 mol of gas $\overline{\mathrm{CO}_{2}}(g)$ is added to the same cylinder keeping the temperature the $\overline{\text { same }}$. What is the new pressure in the cylinder?
$0 \%$ 1. <1 bar
$0 \% 2$. $=1$ bar
$0 \%$ 3. Between 1 and 1.5 bar

## $100 \% 4 . \quad 1.5 \mathrm{bar}$

$0 \%$ 5. Between 1.5 and 2 bar
$0 \%$ 6. $=2$ bar

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## Partial pressures

A mixture consisting of 0.513 g of $\mathrm{H}_{2}(\mathrm{~g})$ and 16.1 g of $\mathrm{N}_{2}(\mathrm{~g})$ occupies 10.0 L at $20.0^{\circ} \mathrm{C}$. Calculate the partial pressures of each gas in atm.
$n_{\mathrm{H}_{2}}=0.254 \mathrm{~mol}, n_{\mathrm{N}_{2}}=0.575 \mathrm{~mol}, n=0.829 \mathrm{~mol}$
Second step: Use the total moles to calculate the total pressure. $P=\frac{n R T}{V}=0.829 \mathrm{~mol} \times 0.0820575 \frac{\mathrm{Latm}}{\mathrm{K} \mathrm{mol}} \times \frac{293.15 \mathrm{~K}}{(10.0 \mathrm{~L}}=1.99 \mathrm{~atm} / /$

## Partial pressures

A mixture consisting of 0.513 g of $\mathrm{H}_{2}(g)$ and 16.1 g of $\mathrm{N}_{2}(g)$ occupies 10.0 L at $20.0^{\circ} \mathrm{C}$. Calculate the partial pressures of each gas in atm

$$
n_{\mathrm{H}_{2}}=0.254 \mathrm{mgl}, n_{\mathrm{N}_{2}}=0.575 \mathrm{mo},=0.829 \mathrm{~mol} P=1.99 \mathrm{~atm}
$$

Final step: Calculate the partial pressures
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$$
\begin{aligned}
& P_{\mathrm{H}_{2}}=x_{\mathrm{H}_{2}} P=\frac{0.25}{(0.820} \times 1.99 \mathrm{~atm}=0.612 \mathrm{~atm} \\
& \stackrel{\rightharpoonup}{P}_{P_{\mathrm{N}_{2}}}^{=}=x_{\mathrm{N}_{2}} P=\frac{0.575}{0.825} \times \underline{1.99 \mathrm{~atm}=1.38 \mathrm{~atm}} \\
& \text { Check: } \\
& \quad P=P_{\mathrm{H}_{2}}+P_{\mathrm{N}_{2}}=0.612 \mathrm{~atm}+1.38 \mathrm{~atm}=1.99 \mathrm{~atm}
\end{aligned}
$$

## Gas law calculations in chemical reactions

$\int \mathrm{C}_{3} \mathrm{H}_{8}(g)$ and $\varliminf_{2}(g)$ are placed in a sealed, rigid container. After the container $\int$ has been heated to $125^{\circ} \mathrm{C}$, the partial pressures are $P_{\mathrm{C}_{3} \mathrm{H}_{8}}=1.00 \mathrm{~atm}$ and $P_{\mathrm{O}_{2}}=1.00 \mathrm{~atm}$. A spark then initiates the complete combustion of the mixture forming $\mathrm{CO}_{2}(\mathrm{~g})$ and $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$. What is the total pressure in the container at the forming $\mathrm{CO}_{2}(\mathrm{~g})$ and $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$.
end of the reaction at $125^{\circ} \mathrm{C}$ ?

Write the balanced chemical equation.


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## Gas law calculations in chemical reactions



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## Gas law calculations in chemical reactions

[TP] $\mathrm{CH}_{4}(g)$ and $\mathrm{O}_{2}(\mathrm{~g})$ are placed in a sealed, rigid container. After the container has been heated to $125^{\circ} \mathrm{C}$, the partial pressures are $P_{\mathrm{CH}_{4}}=1.00 \mathrm{~atm}$
and $P_{\mathrm{O}_{2}}=1.00 \mathrm{~atm}$. A spark then initiates the complete combustion of the mixture, forming $\mathrm{CO}_{2}(\mathrm{~g})$ and $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$. What is the total pressure in the container at the end of the reaction at $125^{\circ} \mathrm{C}$ ? (1) Desuced epu
. 1.50 atm
$100 \%$ 4. 2.00 atm
$0 \% 5 . \quad 2.50 \mathrm{~atm}$
$0 \% 6$. 3.00 atm
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$\square$

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$\mathrm{CH}_{4}(g)+2 \mathrm{O}_{2}(g) \rightarrow \mathrm{CO}_{2}(g)+2 \mathrm{H}_{2} \mathrm{O}(g)$
$1.00 \mathrm{~atm} \mathrm{CH} 4 \rightarrow 1.00 \mathrm{~atm} \mathrm{CO} 2$
$1.00 \mathrm{~atm} \mathrm{O}_{2} \rightarrow 0.500 \mathrm{~atm} \mathrm{CO} 2$
Determine all partial pressures and the total pressure.
e

## Gas law calculations in chemical reactions

$$
\mathrm{C}_{3} \mathrm{H}_{8}(g)+5 \mathrm{O}_{2}(g) \rightarrow 3 \mathrm{CO}_{2}(g)+4 \mathrm{H}_{2} \mathrm{O}(g)
$$

$$
1.00 \mathrm{~atm} \mathrm{O}_{2} \rightarrow 0.600 \mathrm{~atm} \mathrm{CO}
$$

Determine all partial pressures and the total pressure.

|  | $\mathrm{C}_{8} \mathrm{H}_{8}(\mathrm{~g})$ | $\mathrm{O}_{2}(\mathrm{~g})$ | $\mathrm{CO}(g)$ | $\mathrm{H}_{2} \mathrm{O}(g)$ | $P_{\text {total }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Initial | 1.00 | 1.00 | 0 | 0 | 2 |
| Change | -0.200 | -1.00 | +0.600 | +0.800 |  |
| Final | 0.80 | 0 | 0.600 | 0.800 | 2.20 |



42


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43

## Gas law calculations in chemical reactions

$$
\mathrm{CH}_{4}(g)+2 \mathrm{O}_{2}(g) \rightarrow \mathrm{CO}_{2}(g)+2 \mathrm{H}_{2} \mathrm{O}(g)
$$

$$
1.00 \mathrm{~atm} \mathrm{O} \mathrm{O}_{2} \rightarrow 0.500 \mathrm{~atm} \mathrm{CO} 2
$$

Determine all partial pressures and the total pressure.


